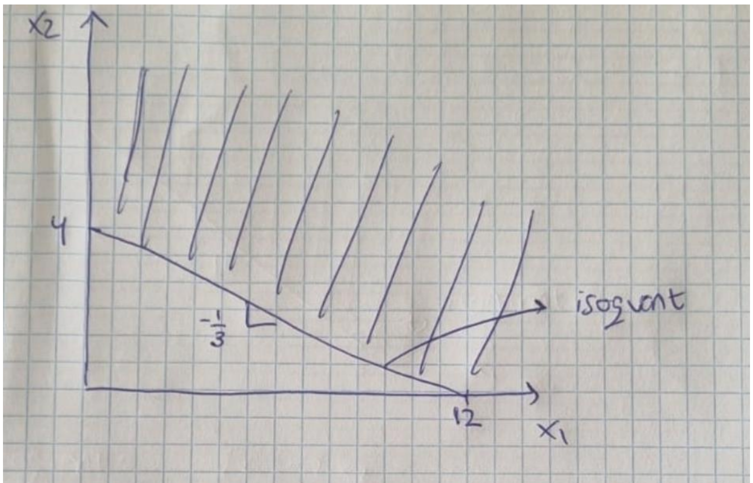
1.1. (1 point)

Note that we can rewrite the production function as $y = \beta x_1 + (1+\gamma)x_2$.

$$TRS = -\frac{\frac{df}{dx_1}}{\frac{df}{dx_2}} = -\frac{\beta}{(1+\gamma)}$$

1.2. (1 point)

The production function is $y = 1x_1 + 3x_2$.

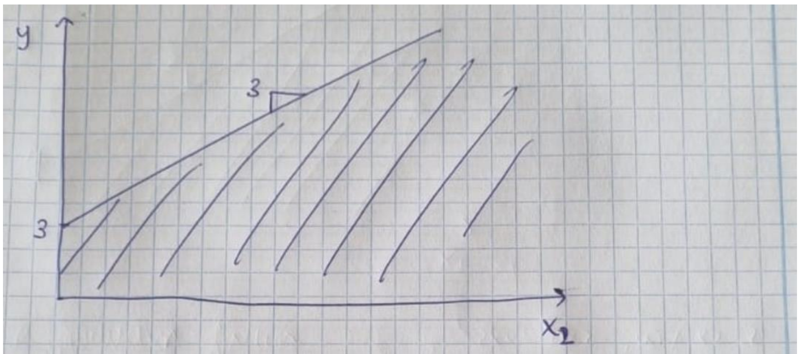


isoquant : $1x_1 + 3x_2 = 12$

$3x_2 = 12 - x_1$

$x_2 = 4 - \frac{1}{3}x_1$

1.3. (1 point)



SR production function : $y = 1x_1 + 3x_2$, w/ $x_1 = 3$

$y = 3 + 3x_2$

2.1. (1 point)

Let $\boldsymbol{w}^t$ be a vector of input prices and $\boldsymbol{x}^t$ be a vector of inputs, in time t. Then the formula for WACM is:

$$\boldsymbol{w}^t\boldsymbol{x}^t \leq \boldsymbol{w}^t\boldsymbol{x}^s, \qquad for\ each\ s, t\ with\ y^s \geq y^t$$

Hence, to test WACM we need the following data over time t:

- Input prices $\boldsymbol{w}^t$
- Input levels $\boldsymbol{x}^t$
- Output levels y^t

2.2. (1 point)

In words, the Envelope theorem explains that when the value of an optimized function changes due to a change in an exogenous variable, only the direct impact of that exogenous variable needs to be considered. This holds true even if the exogenous variable also influences the optimized function indirectly through its effect on an endogenous choice variable.

2.3. (1 point)

The production function implies that the inputs are perfect complements. Hence, to produce any given level of output y, the firm must set $x_1 = x_2 = y$. Otherwise, the smaller input would limit production. Hence, in this case, the conditional factor demands are dictated solely by the desired output y, and not by the costs of the inputs, w_1 and w_2.

2.4. (1 point)

Now the production function implies that the inputs are perfect substitutes. Hence, to produce any given level of output y, the firm will choose the input that is cheapest. If $w_1 < w_2$ it will choose $x_1 = y$ and $x_2 = 0$. Conversely, if $w_2 < w_1$, it will choose $x_1 = 0$ and $x_2 = y$. Hence, in this case, the conditional factor demands may be highly sensitive to the costs of the inputs, w_1 and w_2, as changes in the input price may cause the firm to substitute entirely from one input to the other.

3.1. (1 point)

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 and x2 to reach:

$$x_1 = \frac{m}{p_1}\frac{\alpha}{\alpha + \beta}$$

$$x_2 = \frac{m}{p_2}\frac{\beta}{\alpha + \beta}$$

3.2. (1 point)

$$\frac{dx_1}{dm} = \frac{1}{p_1}\frac{\alpha}{\alpha + \beta}$$

$$\frac{dx_2}{dm} = \frac{1}{p_2}\frac{\beta}{\alpha + \beta}$$

For the Marshallian demand functions x_1 and x_2 to be less steep (with price on the vertical axis and quantity on the horizontal axis) than the Hicksian demand functions h_1 and h_2 we need the income effect to be positive (so that the goods are normal). Hence, we need that:

$$\frac{dx_1}{dm} = \frac{1}{p_1}\frac{\alpha}{\alpha + \beta} > 0$$

$$\frac{dx_2}{dm} = \frac{1}{p_2}\frac{\beta}{\alpha + \beta} > 0$$

For this we need that $\alpha > 0$ and $\beta > 0$.

3.3. (1 point)

MRS: If a consumer increases consumption for good 1, how much does the consumer need to decrease her consumption for good 2 to keep the same utility level. This describes the slope of the indifference curve.

$$MRS = \frac{dx_2(x_1)}{dx_1} = -\frac{\frac{du}{dx_1}}{\frac{du}{dx_2}} = -\frac{0.5x_1^{-0.5}x_2^{0.5}}{0.5x_1^{0.5}x_2^{-0.5}} = -\frac{x_2}{x_1}$$

3.4. (1 point)

1. Write down the Lagrangian for the EMP

2. Take FOCs

3. Solve these FOCs for x1 and x2 (denoted by h1 and h2) to reach

$$h_1 = \left(\frac{p_2}{p_1}\right)^{0.5} \bar{u}$$

$$h_2 = \left(\frac{p_1}{p_2}\right)^{0.5} \bar{u}$$

4. Plug these into the budget constraint to reach

$$e(p_1, p_2, \bar{u}) = 2p_1^{0.5}p_2^{0.5}\bar{u}$$

The expenditure function $e(p_1, p_2, \bar{u})$ represents the minimum expenditures required to reach utility level $\bar{u}$ at prices p_1 and p_2.

3.5. (1 point)

To obtain lambda one needs to take the derivative of the expenditure function towards $\bar{u}$:

$$\frac{de(p_1, p_2, \bar{u})}{d\bar{u}} = \lambda(p_1, p_2) = 2p_1^{0.5}p_2^{0.5}$$

Plugging in for $p_1 = p_2 = 1$ gives us

$$\lambda(p_1, p_2) = 2 * 1^{0.5} * 1^{0.5} = 2$$

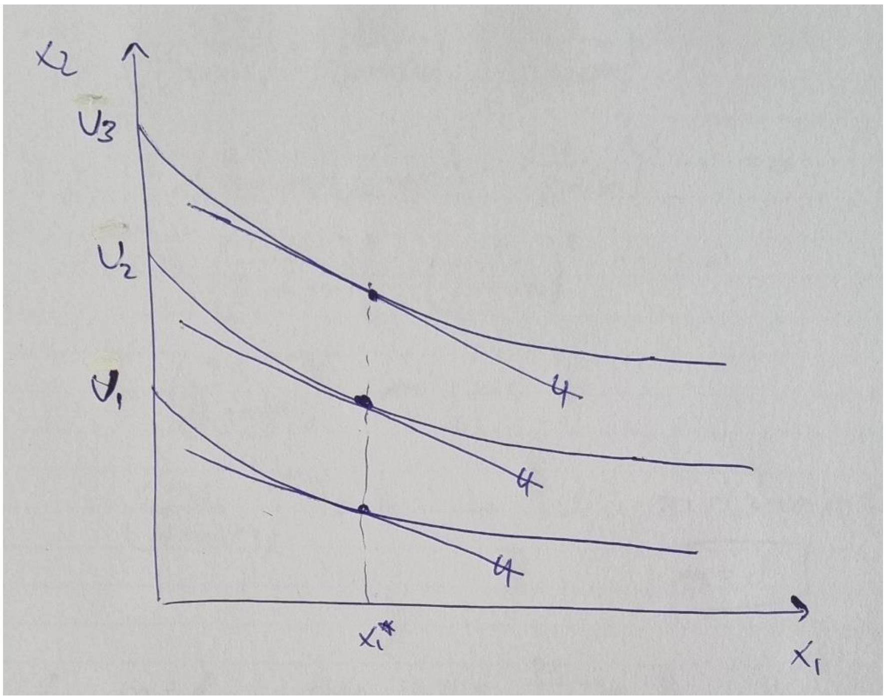
This implies that if we allow the consumer to have one additional util (so we relax the constraint by 1), then the consumer needs to spend 2 additional euros.

4.1. (2 points)

The formula for the indifference curve is:

$x_2 = u - 2\sqrt{x_1}$

And we need to draw this for three levels of u:



Explanation of special feature: The varying levels of u just change the intercept of this indifference curve, where the slope of this indifference curve does not depend upon the levels of u. Hence, the indifference curves are parallel: they are vertical shifts from one to the other.

Explanation of consequence for income effect: since the slope of this indifference curve does not depend upon the level of u, the indifference curves related to the varying levels of u will be tangent to the budget line for varying income levels at the same level of x1. This implies that the income effect for x1 is zero. This is reflected by x_1^* in the figure.

4.2. (1 point)

To find the compensating variation you can follow three steps:

1. Solve the UMP at original prices and income to find indirect utility

$v(p^0, m) = 7$

2. Solve the EMP at new prices and v^0 to find minimum expenditure

$e(p^1, v^0) = 12$

3. Calculate CV by subtracting income in step 1 from minimum expenditure in step 2.

$CV = e(p^1, v^0) - m = 12 - 10 = 2$

4.3. (1 point)

With quasilinear utility, the Marshallian and Hicksian demand coincide.

The compensating variation is the area to the left of the Hicksian demand curve. The change in consumer surplus is the area to the left of the Marshallian demand curve. Since the Hicksian and Marshalian demand coincide, the compensating variation and change in consumer surplus are the same.

5.1. (2 points)

There are two conditions for long run equilibrium:

$Y(p) = X(p)$

$\pi_i = 0 \quad for\ each\ firm\ i$

1. We derive the firms' supply function $y_i(p)$ for each firm i.

$mc_i(y) = \frac{dc_i(y)}{dy} = y,$

and since supply curve is $mc_i(y) = p$

we have that $y_i(p) = p$.

2. We derive market supply, which is the sum over all firms m.

$Y(p) = \sum_{i=1}^{m} y_i(p) = \sum_{i=1}^{m} p = mp.$

3. We use the first condition to find equilibrium price and firm supply in terms of number of firms m.

$$mp = 60 - 5p$$

$$p = \frac{60}{m+5}$$

$$y_i(p) = p = \frac{60}{m+5}$$

4. We use the second condition to find the number of firms m so that profits are zero.

$$\pi_i = py_i(p) - c_i(y) = 0$$

$$\pi_i = \left(\frac{60}{m+5}\right)^2 - 0.5\left(\frac{60}{m+5}\right)^2 - 8 = 0$$

$$0.5\left(\frac{60}{m+5}\right)^2 = 8$$

$$\left(\frac{60}{m+5}\right)^2 = 16$$

$$\frac{60}{m+5} = 4$$

$$m = 10$$

Hence, in the long run there will be 10 active firms in this perfect competitive market.

6.1.(1 point)

For a firm in a perfectly competitive market the price is fixed, and so the marginal revenue is equal to that fixed price. Hence, given the fixed price is positive, the marginal revenue is positive.

For a monopolist the marginal revenue is typically lower than the price, as a monopolist typically needs to lower the price when it wants to sell more. Now when the demand of the monopolist is inelastic, the monopolist needs to lower the price a lot as to sell more units, which can make the marginal revenue negative.

6.2.(1 point)

The mark-up of a monopolist is defined as the difference between the price and the marginal cost, hence as $p - MC$.

The following formula can be derived from the FOC of the monopolist:

$$\frac{p - MC}{p} = -\frac{1}{\epsilon}$$

Where ϵ is the elasticity of demand. It is given that $\epsilon = -3$ and $MC = 10$, so that:

$$\frac{p - 10}{p} = \frac{1}{3}$$

Hence, we know that $p = 15$ and the markup is $15 - 10 = 5$.

The markup may be used as a measure of market power since it reflects the ability to set the price above marginal costs, and hence the ability to make profits (over the last units sold). In extreme, a perfect competitive firm has no market power, as the markup is zero with $p = MC$.